Engineering Notes

Cost Minimization of a Space System by Multiple Launchings

Ralph W. Allen*
Northrop Ventura, Newbury Park, Calif.

Nomenclature

a = auxiliary and ground support equipment (GSE) facility cost exponent on W_G (order of 2)
b = exponent between 0.8 and 1.0 (see W_G)
c_D = development cost factor, \$/lb^r, Eq. (6)
c_F = auxiliary and GSE facility cost factor, Eq. (7)
c_L = launching cost coefficient, cost per launch, \$
c_V = fabrication cost coefficient, \$/lb gross launch weight
K = weight coefficient, probably 15 to 20 (see W_G)
n = number of launchings, W_{PR}/w_P
n_{opt} = computed n for minimum system cost
p = failure probability, single launching
r = development cost exponent on W_G, ≤ 1; Eq. (6)
W_G = gross weight at launch, $Kw_P{}^b + w_P$
w_P = payload weight per booster launch, lb
W_{PD} = total desired system payload weight, lb
W_{PR} = total payload weight needed to put W_{PD} into space, lb

Approximate Analysis and Examples

The gross payload required W_{PR} at the launch site to put a given total payload W_{PD} into space by n launches of equal payloads w_P is

$$W_{PR} = nw_P = W_{PD} + \text{losses} \tag{1}$$

For a given single-launch probability of failure p, the expected loss on the first launch is $w_P p$, which must be made up in subsequent launches, so that the total losses for the n launches required to achieve W_{PD} leads to

$$W_{PR} = nw_P = W_{PD} + w_P p(1 + p + p^2 + p^3 + \ldots p^n) \tag{2}$$

or

$$w_P = W_{PD}/[n + p(1 - p^n)/(1 - p)] \tag{3}$$

To a first approximation, gross weight per vehicle can usually be related to payload weight by

$$W_G = Kw_P{}^b + w_P \tag{4}$$

where b may range from 0.8 to 1.0 and K may be 15 to 20, depending on factors such as design efficiency, specific impulse, and mission. If (4) is substituted into (3) and simplified by letting $b = 1$,

$$W_G = W_{PD}(1 + K)/[n + p(1 - p^n)/(1 - p)] \tag{5}$$

Total system fabrication and development costs are usually related to gross weight in a manner that might be approximated by

$$\text{vehicle system cost} = nW_G c_V + W_G{}^r c_D \tag{6}$$

The cost of launching the system is assumed to be nc_L, where c_L is the cost per launch. The amortized capital cost of auxiliary ground support equipment and facilities generally increases with gross weight by a parameter $c_F W_G{}^a$, where the cost factor c_F depends on facility type and sophistication and the exponent a may be of the order of 2. Adding these factors to (6) gives

$$\text{total cost} = nW_G c_V + W_G{}^r c_D + c_F W_G{}^a + nc_L \tag{7}$$

Total costs can now be estimated by substituting (5) into (7). For all examples of this solution discussed here, $W_{PD} =$ 10,000 lb, $c_V =$ \$50/lb, $c_L =$ \$$10^6$, $K = 20$, and, of course, $b = 1$.

Figure 1 shows some examples for developed systems ($c_D = 0$), with the inputs and other assumed values as shown. For curves B, C, and D, all with $p = 0.5$, but with decreasing facility cost factor c_F, the minimum costs occur at $n = 7$, 6, and 5, respectively; i.e., the optimum number of launches is not very sensitive to facility cost for the system considered when $p = 0.5$. This small c_F effect on n_{opt} is further illustrated by the cross plot in Fig. 2. Comparison

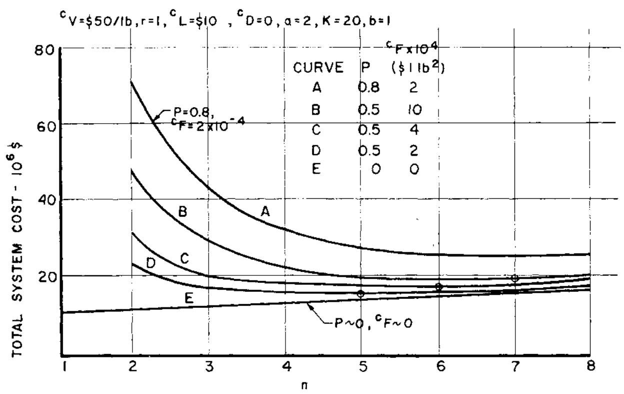

Fig. 1 Examples of variations in total cost with number of launchings, excluding development costs, under various conditions.

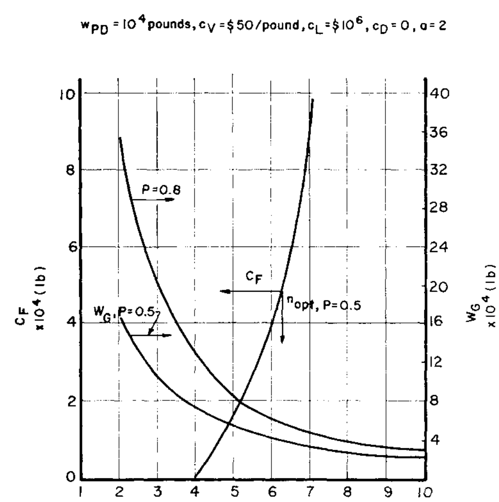

Fig. 2 Relations between auxiliary facility cost and optimum number of launchings ($p = 0.5$) and between vehicle gross weight and number of launchings.

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* Principal Scientist. Member AIAA.

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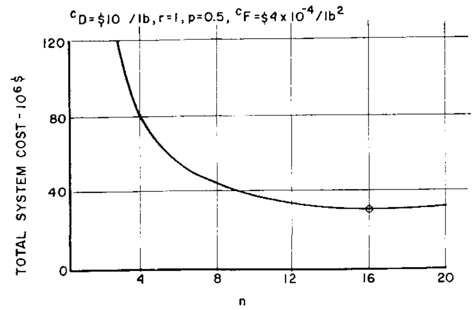

Fig. 3 Example of variation in total system cost with number of launchings including development costs.

of curve A ($p = 0.8$) with curve C in Fig. 1 shows that the optimum n decreases as p decreases; of course, when p and c_F become very small for a developed system, a single launch by a large booster will be best (curve E). Figure 2 also shows the variation of vehicle gross weight with number of launches for both $p = 0.5$ and $p = 0.8$. Comparison of these curves shows that if 5 launches (near minimum cost) were used at $p = 0.5$, 8 launches would be required to put the same total payload up with $p = 0.8$.

Figures 1 and 2 assumed no development cost. If a development cost c_D = \$1000/lb (with $r = 1.0$) is added to the other conditions represented by curve C of Fig. 1, the curve of Fig. 3 results. The optimum number of launches has tripled, from 5 in Fig. 1 to 16 in Fig. 3. Of course, the effect would be smaller for r somewhat less than unity (which would probably be more realistic), and furthermore, r should tend to decrease as n goes up (due to mass production benefits), so that the sample comparison is somewhat exaggerated.

Concluding Remarks

These simple examples have indicated that under certain conditions it would be better to boost a given payload into space by dividing it into several parts boosted separately, as one might have expected. Of course, only launch-system costs have been considered here; if mission constraints for manned vehicles (e.g., for re-supply missions) were imposed, there would probably be more distinct limits on n. Further, for payloads to be assembled in space, there could arise a weight penalty for assembly problems at large n. Finally, for manned missions, a very low failure probability p will necessarily be sought; this leads to low n_{opt}. However, it remains that a rather simple analysis might be used to assess the importance of various factors on optimum vehicle size for multiple launchings to put a given total payload into space, or for providing a multi-purpose spacecraft booster system.

Elastomeric Seals in Hard Vacuum

Ted R. Carrell* and Harold Blair†
Parker Seal Company, Culver City, Calif.

EFFECTIVE sealing against the "hard" vacuum of interplanetary space has been one of the problems facing designers of advanced vehicles. At first, use of elastomeric (rubber) seals in space applications seemed limited, since early studies indicated that hard vacuums would cause conventional elastomers to volatilize and evaporate.

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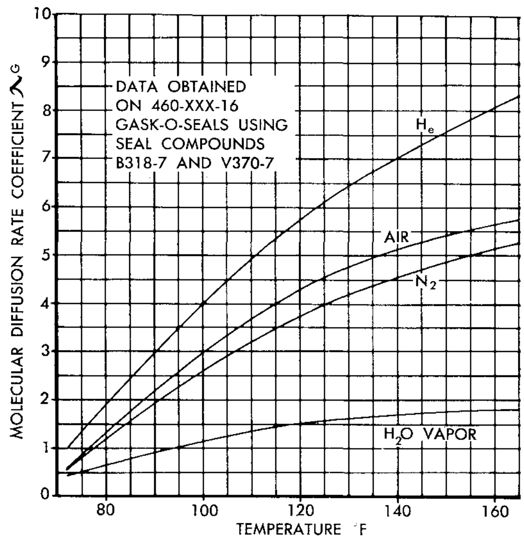
* Manager, Engineering Services.

† Chief Engineer, Development.



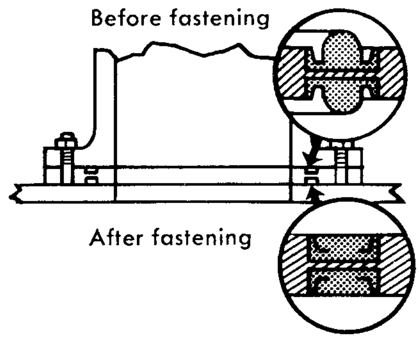


Fig. 1 Vacuum molecular diffusion rate coefficient of various gases as a function of temperature.

However, assuming that better elastomeric polymers and special seal designs might change the viewpoint toward elastomers, the authors conducted a test program to provide data on hard vacuum elastomeric seals for specific space projects. The results indicate a much greater design flexibility for engineers in the application of elastomer seals for vacuum than was generally supposed, and the possibility of a mathematical formula for determining leak rates of various gases with such seals.

Hard vacuum (vacuums below the level of 10^{-6} mm Hg, torr) affects elastomeric materials in two general ways. First, evaporation of the material or of a volatile component of the material may occur. Second, high vacuum may cause alteration of the seal structure affecting its mechanical characteristics and other physical properties. The rates of these reactions can be radically affected by thermal extremes.

However, a research program had already provided data showing that, although certain rubbers volatilize rapidly from 10^{-7} to 10^{-8}, a specially formulated elastomeric material was unaffected at vacuums of 10^{-9} torr.[1]

There was also evidence to support the supposition that the percentage of weight loss depends on geometry, for only those monomers at the surface were removed. Studies performed on meteorites verify the concept of progressive surface decomposition of macromolecules of polymeric materials in hard vacuums. This indicated that careful design of seal geometry might help minimize vacuum erosion.

The program was based on the use of specially compounded seal materials in a "controlled confinement" (Gask-O-Seal)‡ design configuration. This type of seal has been widely used for many years in aircraft and missile applications. Basically, it is a grooved metal retainer containing a molded-in-place special-configuration rubber seal element. Typical

‡ Parker Seal Company Trademark.